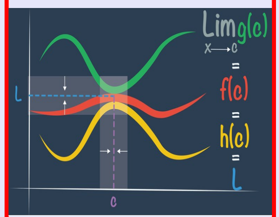
Calculus I

Lecture 5



Feb 19-8:47 AM

Class QZ 1

Given $f(x) = 3x^2 - 5x - 8$

1) Y-Int $\quad f(0) = 3(0)^2 - 5(0) - 8 = -8 \Rightarrow \boxed{(0,-8)}$ ✓

2) x-Int. $\quad f(x) = 0$

$3x^2 - 5x - 8 = 0$

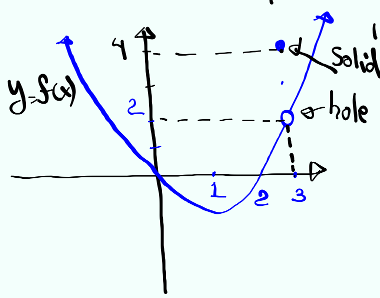
$(3x - 8)(x + 1) = 0$

$x = \frac{8}{3} \qquad x = -1$

$\Rightarrow \boxed{\left(\frac{8}{3}, 0\right), \; (-1, 0)}$

Aug 29-8:18 AM

Consider the graph below



1) $\lim_{x \to 3^+} f(x) = 2$

2) $\lim_{x \to 3^-} f(x) = 2$

3) $\lim_{x \to 3} f(x) = 2$

4) $f(3) = 4$

when $\lim_{x \to a} f(x) = f(a) \Rightarrow f(x)$ is Continuous at $x = a$

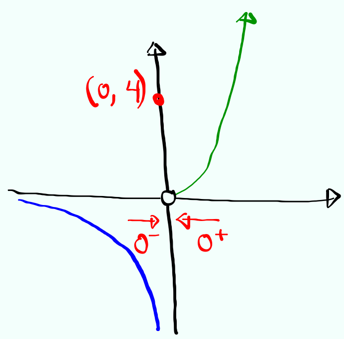
when $\lim_{x \to a} f(x) \neq f(a) \Rightarrow f(x)$ is not Continuous at $x = a$.

Sep 3-7:31 AM

Consider

$$f(x) = \begin{cases} \frac{1}{x} & \text{if } x < 0 \\ 4 & \text{if } x = 0 \\ x^2 & \text{if } x > 0 \end{cases}$$

Piece-wise Function



1) $\lim_{x \to 0^+} f(x) = 0$

2) $\lim_{x \to 0^-} f(x) = -\infty$

3) $\lim_{x \to 0} f(x) =$ D.N.E.

4) $f(0) = 4$

5) Is $f(x)$ continuous at $x = 0$? Explain.

No

Since $\lim_{x \to 0} f(x) \neq f(0)$

6) Domain $\mathbb{R}$, $(-\infty, \infty)$

7) Range $(-\infty, 0) \cup (0, \infty)$

$y \neq 0$

Sep 3-7:36 AM

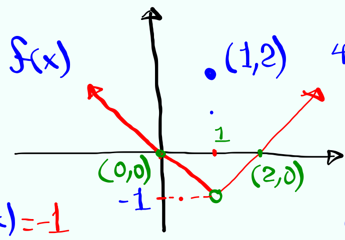
Given $y = |x|$ $+1$ -1

$$f(x) = \begin{cases} |x-1| - 1 & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$$

1) $f(2) = |2-1| - 1 = 0$

2) $f(0) = |0-1| - 1 = 0$

3) Draw $f(x)$



4) $\lim\limits_{x \to 1^-} f(x) = -1$

5) $\lim\limits_{x \to 1^+} f(x) = -1$

6) $\lim\limits_{x \to 1} f(x) = -1$

7) Is $f(x)$ cont. at $x = 1$? Explain. NO $\lim\limits_{x \to 1} f(x) \neq f(1)$

8) Domain $(-\infty, \infty)$

9) Range $(-1, \infty)$

Sep 3-7:48 AM

How to evaluate the limit:

1) Plug it in, and evaluate.

$$\lim_{x \to 1} \left(\sqrt[3]{x} + 5x\right) = \sqrt[3]{1} + 5(1) = 1 + 5 = \boxed{6}$$

$$\lim_{x \to -1} \frac{x^2 + 3x}{x - 2} = \frac{(-1)^2 + 3(-1)}{-1 - 2} = \frac{1 - 3}{-3} = \frac{-2}{-3} = \boxed{\frac{2}{3}}$$

$$\lim_{x \to \pi} (\sin x - \cos x) = \sin \pi - \cos \pi = 0 - (-1) = \boxed{1}$$

Sep 3-7:58 AM

If step 1 gives us an issue $\frac{0}{0}$, then we simplify by factoring, by rationalizing, or by other forms.

$$\lim_{x \to 1} \frac{x^3 - 1}{x^2 - 1} = \frac{1^3 - 1}{1^2 - 1} = \frac{1-1}{1-1} = \frac{0}{0}$$

$A^3 - B^3 \qquad (A-B)(A^2 + AB + B^2)$

$$\lim_{x \to 1} \frac{x^3 - 1}{x^2 - 1} = \lim_{x \to 1} \frac{\cancel{(x-1)}(x^2 + x + 1)}{(x+1)\cancel{(x-1)}}$$

$A^2 - B^2 \qquad (A+B)(A-B)$

$$= \lim_{x \to 1} \frac{x^2 + x + 1}{x + 1} = \frac{1^2 + 1 + 1}{1 + 1} = \boxed{\frac{3}{2}}$$

Sep 3-8:05 AM

Evaluate $\lim_{x \to -2} \frac{x^2 + 5x + 6}{x^2 - 4} = \frac{(-2)^2 + 5(-2) + 6}{(-2)^2 - 4} = \frac{0}{0}$

I.F.

$$\lim_{x \to -2} \frac{x^2 + 5x + 6}{x^2 - 4} = \lim_{x \to -2} \frac{(x+3)\cancel{(x+2)}}{(x-2)\cancel{(x+2)}} = \lim_{x \to -2} \frac{x+3}{x-2}$$

$$= \frac{-2+3}{-2-2} = \boxed{\frac{-1}{4}}$$

Sep 3-8:10 AM

Evaluate $\lim_{x \to \frac{1}{3}} \dfrac{\frac{1}{x} - 3}{3x - 1} = \dfrac{\frac{1}{\frac{1}{3}} - 3}{3\left(\frac{1}{3}\right) - 1} = \dfrac{3-3}{1-1} = \dfrac{0}{0}$

I.F.

$$\lim_{x \to \frac{1}{3}} \frac{\frac{1}{x} - 3}{3x - 1} = \lim_{x \to \frac{1}{3}} \frac{x \cdot \frac{1}{x} - x \cdot 3}{x(3x - 1)} = \lim_{x \to \frac{1}{3}} \frac{\overset{-1}{\cancel{1 - 3x}}}{x\cancel{(3x - 1)}}$$

LCD $= x$

$$= \lim_{x \to \frac{1}{3}} \frac{-1}{x} = \frac{-1}{\frac{1}{3}} = \boxed{-3}$$

Recall

$$\frac{a - b}{b - a} = -1$$

Sep 3-8:16 AM

Evaluate $\lim_{x \to 4} \dfrac{\sqrt{x} - 2}{x - 4} = \dfrac{\sqrt{4} - 2}{4 - 4} = \dfrac{2-2}{4-4} = \dfrac{0}{0}$ I.F.

Hint: Rationalize the numerator

$$\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4} = \lim_{x \to 4} \frac{\overset{A\ -B}{(\sqrt{x} - 2)} \cdot \overset{A\ +B}{(\sqrt{x} + 2)}}{(x - 4) \cdot (\sqrt{x} + 2)}$$

$$= \lim_{x \to 4} \frac{\overset{A^2 - B^2}{(\sqrt{x})^2 - (2)^2}}{(x - 4)(\sqrt{x} + 2)}$$

$$= \lim_{x \to 4} \frac{\cancel{x - 4}}{\cancel{(x - 4)}(\sqrt{x} + 2)} = \lim_{x \to 4} \frac{1}{\sqrt{x} + 2}$$

$$= \frac{1}{\sqrt{4} + 2} = \boxed{\frac{1}{4}}$$

Sep 3-8:22 AM